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Glueball mass spectra for supergravity duals of noncommutative gauge theories

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Abstract: We derive the glueball masses in noncommutative super Yang-Mills theories in four dimensions via the dual supergravity description. The spectrum of glueball masses is discrete due to the noncommutativity and the glueball masses are proportional to the noncommutativity parameter with dimension of length. The mass spectrum in the WKB approximation closely agrees with the mass spectrum in finite temperature Yang-Mills theory.

Keywords: Non-Commutative Geometry, AdS-CFT Correspondence.

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1. Introduction

In recent years, it has been possible to investigate aspects of the large N noncommutative gauge theories in the spirit of the AdS/CFT [1−3] correspondence. Noncommutative gauge theories arise as a certain low-energy limit of string theory in Neveu-Schwarz-Neveu-Schwarz (NS-NS) B-field background [4]. Supergravity duals of large N noncommutative gauge theories with maximal supersymmetry have been constructed as the decoupling limits of D-brane solutions with NS-NS B fields [5, 6]. Noncommutative gauge theories are intriguing dynamical systems which exhibit rich features such as gauge invariance, nonlocality and UV/IR mixing. These supergravity solutions have been used to investigate qualitative aspects of nonperturbative gauge theories [15−20]. Since noncommutativity introduces a new physical scale to the theories, it modifies the Wilson loop behavior. If noncommutativity effects are large, then they exhibit area law [13, 22, 14]. Supergravity duals of noncommutative gauge theories with less than maximal supersymmetry have also been constructed [7, 32]. The behavior of the Wilson loops in $\mathcal{N} = 1$ NCSYM theory has been investigated from a deformation of the Maldacena–Nùñez solution, which is proposed as supergravity duals of $\mathcal{N} = 1$ NCSYM theory. The quark-antiquark potential via the Wilson loop gives a same behavior as ordinary $\mathcal{N} = 1$ super Yang–Mills theory in the IR region, although the UV physics give a different behavior. The β-function in the $\mathcal{N} = 1$ NCSYM theory has also been computed and the β-function of the NCSYM theory coincides with the ordinary one [32].

It is well known that noncommutative gauge theories have no local gauge invariant operators. Nevertheless there are non-local gauge invariant operators which are the Fourier transform of local operators attached to open Wilson lines [8, 9]. It seems to indicate that the supergravity fields act as sources of such kind of gauge invariant operators [10−12]. The fact that supergravity fields do not depend on the noncommutative coordinates makes it easier to obtain the gravity fields that are dual to such kind of gauge theory operators.

Supergravity solutions have been used to study qualitative aspects of non-perturbative gauge theories not only quark confinement, but also chiral symmetry breaking, renormalization group flow, binding energy of the baryon and glueball mass spectrum. A discrete glueball spectrum with a finite gap can be derived by compactification of the dual supergravity models. The radius of the compactifying circle provides the ultraviolet cut-off scale and the glueball masses are measured in unit of the compactification radius. The ratio of the glueball masses is a fairly good quantitative agreement with lattice data [23 − 26].

In this paper, we study some nonperturbative aspects of noncommutative Yang-Mills (NCYM) theories by focusing on evaluating mass spectra of the glueballs. Since NCYM theories have an intrinsic physical scale, there is a possibility that the physical scale reflects the discrete mass spectra without any compactifications. The paper is organized as follows. In section 2, we evaluate 0^{++} glueball masses in noncommutative super Yang-Mills (NCSYM) theory in four dimensions by solving the wave equations for dilaton in the dual supergravity background. The mass eigenvalues can be determined approximately via the Wentzel-Kramers-Brillion (WKB) analysis. 0^{++} glueball masses in NCSYM theory in a constant self-dual of gauge field background are also evaluated using the dual supergravity description. In section 3, we evaluate 1^{--} glueball masses in NCSYM theory in four dimensions by solving the wave equations for antisymmetric tensor field in the dual supergravity background. All the results are compared with the glueballs masses in finite temperature Yang-Mills theories from supergravity computation and lattice computations. Section 4 is devoted to conclusions and discussions.

2. The 0^{++} glueball masses in noncommutative gauge theory

2.1 Glueball masses in NCSYM theory

We begin with the D3 brane solution in a NS-NS B-field background in the near horizon limit [21, 22]:

$$ds^2 = \alpha' R^2 \left[u^2 \left\{ -dx_0^2 + dx_1^2 + \hat{h}\,(dx_2^2 + dx_3^2) \right\} + \left(\frac{du^2}{u^2} + d\Omega_5^2 \right) \right], \tag{2.1}$$

where

$$\hat{h}^{-1} = 1 + a^4 u^4 \,. \tag{2.2}$$

Here we assume that the NS-NS B field has the non-vanishing component of B_{23}. In order to obtain NCSYM theory we should take the B-field to infinity in the near horizon limit as $B\alpha' = fixed$. The noncommutativity parameter a is related to the rescaling B-field $\widetilde{B}_{23}$ as $\widetilde{B}_{23} = \frac{\alpha' R^2}{1+a^4u^4}$.

The 0^{++} glueball masses can be derived from the 2-point function of the dimension 4 scalar operators $\mathcal{O}_4 = trF^2$. The scalar operators $\mathcal{O}_4$ couples to the real part of a complex massless scalar field that consists of the dilaton and the Ramond-Ramond (R-R) scalar field. When we evaluate the 0^{++} glueball masses, we have to solve the classical equation of motion of the massless dilaton in the supergravity background [23, 26]. Consider the wave equation for the dilaton:

$$\partial_\mu \left\{ e^{-2\phi} \sqrt{g} g^{\mu\nu} \partial_\nu \phi \right\} = 0 \,. \tag{2.3}$$

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Under the metric of (2.1) the dilaton equation (2.3) is given by:

$$\partial_u \left[u^5 \partial_u \rho \right] + u(k_0^2 - k_1^2)\rho - (1 + a^4 u^4) u (k_2^2 + k_3^2)\rho = 0 \,. \tag{2.4}$$

In deriving this equation, we assume that the dilaton ϕ has the plane wave form $\phi = e^{ik\cdot x}\rho(u)$. The glueball mass M^2 is equal to $-k^2$. In order to take in the effects of the noncommutativity to the wave equation, we choose a particular momentum $k^\mu = (\frac{M}{\sqrt{1-\beta^2}}, 0, \frac{\beta M}{\sqrt{1-\beta^2}}, 0)$ that is given by *the Lorentz boost* of the rest frame momentum $k^\mu = (M, 0, 0, 0)$. In other words, we consider the dilaton equation in the "moving" frame with the velocity β in unit of the light velocity [13, 15, 32]. Then the equation (2.4) becomes

$$\partial_u \left[u^5 \partial_u \rho \right] + \frac{M^2}{1-\beta^2} u [1 - \beta^2 (1 + a^4 u^4)]\rho = 0 \,. \tag{2.5}$$

When we change the variable to $y = u^2$, the equation takes the form

$$\partial_y \left[y^3 \partial_y \rho \right] + \frac{M^2}{4(1-\beta^2)} [1 - \beta^2 (1 + a^4 y^2)]\rho = 0 \,. \tag{2.6}$$

Since the differential equation (2.6) has singularities at $y = 0$ and $y \to \infty$, we rewrite the equation (2.6) by using a new variable $a^2 y = e^z$. Then we have

$$\partial_z \left[e^{2z} \partial_z \rho \right] + \frac{M^2 a^2}{4} e^z [1 - \gamma e^{2z}]\rho = 0 \,, \tag{2.7}$$

with $\gamma \equiv \frac{\beta^2}{1-\beta^2}$. For a definition of the new function $\rho = e^{-z}\psi$ we can obtain the Schrödinger-type equation as

$$\psi'' + V\psi = 0 \,, \tag{2.8}$$

where $'$ denotes the differentiation with respect to the variable z. The explicit form of the potential for the Schrödinger equation (2.8) is given by

$$V = -\frac{1}{4} M^2 a^2 \gamma e^{-z} (e^z - a^2 \lambda_+)(e^z - a^2 \lambda_-) \,, \tag{2.9}$$

where

$$\lambda_\pm = -\frac{2}{M^2 a^4 \gamma} \left\{ 1 \pm \sqrt{1 + \frac{M^4 a^4 \gamma}{4}} \right\} . \tag{2.10}$$

This potential has the turning points at $z = \ln(a^2 \lambda_-)$. We shall evaluate the mass spectrum within the semiclassical WKB approximation. The WKB approximation for this potential gives

$$\begin{aligned} \left(n + \frac{1}{2} \right) \pi &= \int_{-\infty}^{\ln \lambda_-} dz \sqrt{V} \\ &= \sqrt{\frac{1}{4} M^2 a^4 \gamma} \int_0^{\lambda_-} dy \sqrt{\frac{(y - \lambda_+)(\lambda_- - y)}{y^3}} \,, \end{aligned} \tag{2.11}$$

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where n denotes the integer. By substituting the variable $y = \lambda_- t$ into the last line of eq. (2.11), we can rewrite the WKB integral as

$$\left(n+\frac{1}{2}\right)\pi = \sqrt{\frac{1}{4}M^2 a^4 \gamma\,(-\lambda_+)}\int_0^1 dt\, t^{-3/2}(1-t)^{1/2} \times \\ \times \left\{1 - \frac{1}{2}\frac{\lambda_-}{\lambda_+}t - \sum_{m=2}^{\infty}\frac{(2m-3)!!}{2^n}\left(\frac{\lambda_-}{\lambda_+}t\right)^m\right\}. \tag{2.12}$$

In deriving eq. (2.12), we have expanded the expression $\sqrt{1-\frac{\lambda_-}{\lambda_+}t}$ in the Taylor's series by taking account of the fact that $0 < -\frac{\lambda_-}{\lambda_+}t < 1$. The magnitude of the parameter $\frac{\lambda_-}{\lambda_+}$ takes values smaller than 0.1 for the noncommutativity parameter $a \sim M^{-1}$ and the velocity $\beta < 0.8$. Even though the noncommutativity parameter a takes large value such as $a \sim 10^4 M^{-1}$, the parameter $\frac{\lambda_-}{\lambda_+}$ takes such a sufficiently small values in the low velocity $\beta < 0.014$. Hereafter we restrict our computation within the low velocity region where the perturbative analysis is appropriate. The right hand side of the expression (2.12) is given as the function of the dimensionless quantity $(Ma)^4$. The glueball masses are obtained by solving the WKB approximation (2.12) with respect to the quantity $(Ma)^4$ after carrying out the integration. Up to the leading order in the parameter $\frac{\lambda_-}{\lambda_+}$ we obtain

$$(Ma)^4 = \frac{(2n+1)^2(2n-1)(2n+3)}{\gamma}. \tag{2.13}$$

Here we have utilized a regularization based on the analytic continuation for Euler's integral of the first kind: $\int_0^1 dt\, t^{-3/2}(1-t)^{1/2} \equiv \lim_{p\to -\frac{1}{2}} B(p,\, \frac{3}{2}) = -\pi$, where $B(p,\, q)$ denotes the Euler's beta function. The mass spectrum for 0^{++} glueball is given by

$$M_{(L)}^{0^{++}} = \frac{1}{a}\sqrt[4]{\frac{(2n+1)^2(2n-1)(2n+3)}{\gamma}}, \tag{2.14}$$

The glueball masses (2.14) takes real numbers for positive integer $n = 1, 2, 3, \ldots$. Notice that the glueball masses are proportional to the inverse of the noncommutativity parameter a. When we take the commutative limit $a \to 0$, then the masses do not take the discrete values.

Up to the subleading order in the parameter $\frac{\lambda_-}{\lambda_+}$ of the WKB approximation leads the mass spectrum for 0^{++} glueball:

$$M_{(L+SL)}^{0^{++}} = \frac{1}{a}\sqrt[4]{\frac{4}{81}\frac{f_+(n)}{\gamma}}, \tag{2.15}$$

where $f_\pm(n)$ denotes some function of the positive integer n whose explicit form is given by

$$f_\pm(n) = 512n^4 + 1024n^3 + 96n^2 - 416n + 8 \pm \\ \pm 8(16n^2 + 16n - 11)\sqrt{(2n+1)^2(4n^2+4n-2)}. \tag{2.16}$$

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state	WKB(up to leading)	WKB(up to subleading)
0^{++}	1 (input)	1 (input)
0^{++*}	1.85	1.89
0^{++**}	2.64	2.73
0^{++***}	3.43	3.56

Table 1: Masses of the 0^{++} glueball in NCYM_4.

state	$NCYM_4$ (WKB)	QCD_4 (WKB) [26]	QCD_4 (Lattice) [28, 29]
0^{++}	1 (input)	1 (input)	1 (input)
0^{++*}	1.89	1.62	1.75 ($\pm$0.17)
0^{++**}	2.73	2.24	-
0^{++***}	3.56	2.82	-

Table 2: Masses of the 0^{++} glueball from supergravity and lattice QCD.

The glueball masses $M^{0^{++}}_{(L+SL)}$ take the positive real eigenvalues, while the other choice of glueball masses $\widetilde{M}^{0^{++}}_{(L+SL)} \equiv \frac{1}{a}\sqrt[4]{\frac{4}{81}\frac{f_-(n)}{\gamma}}$ take complex eigenvalues. The states for 0^{++} glueball with the masses $\widetilde{M}^{0^{++}}_{(L+SL)}$ are unstable. As will be seen later, however, these unstable states are avoidable by virtue of introducing a constant self-dual gauge field background.

As was expected that the glueball masses are given in units of the noncommutativity parameter a. The noncommutativity parameter in the NCYM theory plays a similar role to a compactification radius in the Yang-Mills theory compactified on a circle, or the temperature in the Yang-Mills theories at finite temperature, where the temperature is proportional to the inverse of the compactification radius [23, 26, 27]. We should notice that the spatial noncommutativity make it possible to obtain discrete mass spectrum in the Yang-Mills theory without any compactifications.

Although the expression (2.15) is a bit complicated, there is little difference between the ratios of the 0^{++} glueball masses up to the leading order and the subleading order. The ratios of the 0^{++} glueball masses $M^{0^{++}}_{(L+SL)}$ obtained by solving the dilaton wave equation in the WKB approximation are listed in table 1.

The glueball masses also depend on the boost parameter γ, besides the noncommutativity parameter a. When we evaluate the glueball masses M in the rest flame with the boost parameter $\gamma = 0$, then the glueball can not take the discrete mass spectrum with a finite gap. This is consistent with the fact that the effects of the noncommutativity are taken in by the moving frame. The ratio of the masses does not depend not only on the noncommutativity parameter a, but also on the boost parameter γ, which is a dimensionless parameter. This fact is not an accident. We can regard the WKB integral (2.12) as an algebraic equation for the variable $M^4a^4\gamma$. If we can solve the algebraic equation, then we have the variable $M^4a^4\gamma$ as a function of the integer n. therefore, the glueball masses are also a function of the integer $F(n)$ as $M = a^{-1}\gamma^{-1/4}F(n)$. Although the masses depend on the noncommutativity parameter a and the boost parameter γ, the ratios of the masses are

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independent of both parameters. Comparison of 0^{++} glueball masses in finite temperature Yang-Mills theory in four dimensions from supergravity, besides of the lattice QCD results in four dimensions is shown in table 2. From table 2 we find that the difference between the supergravity or lattice results in the QCD and the supergravity ones in the NCYM theory is small.

2.2 Glueball masses in NCSYM theory in a constant self-dual background

In this subsection we evaluate the glueball masses in the four dimensional NCSYM theory in a constant self-dual gauge field background using the dual supergravity description. Its supergravity dual is known as a limit of superposition of D3-brane and D(-1)-brane (D-instanton) backgrounds [21, 22]. The metric for the supergravity solution in the near horizon limit is:

$$ds^2 = \alpha' R^2 \left(1 + \frac{q}{R^4 u^4}\right)^{1/2} \left[u^2 \left\{-dx_0^2 + dx_1^2 + \hat{h}\,(dx_2^2 + dx_3^2)\right\} + \left(\frac{du^2}{u^2} + d\Omega_5^2\right)\right], \tag{2.17}$$

where

$$\hat{h} = \frac{1}{1 + Ha^4u^4},$$

with $H = 1 + \frac{q}{R^4u^4}$. Here q denotes the D-instanton density.

Under the metric of (2.17) we obtain the wave equation for the dilaton $\phi = e^{ik\cdot x}\rho(u)$

$$\partial_u \left[u^5 \partial_u \rho\right] + \frac{M^2}{1-\beta^2} u[1 - \beta^2(1 + Ha^4u^4)]\rho = 0\,, \tag{2.18}$$

with a particular momentum $k^\mu = (\frac{M}{\sqrt{1-\beta^2}}, 0, \frac{\beta M}{\sqrt{1-\beta^2}}, 0)$. Changing the variable to $z = 2\ln(au)$ we have

$$\partial_z \left[e^{2z} \partial_z \rho\right] + \frac{M^2a^2}{4} e^z [1 + \gamma H e^{2z}]\rho = 0\,, \tag{2.19}$$

where $\gamma = \frac{\beta^2}{1-\beta^2}$. For a redefinition of a function $\rho = e^{-z}\psi$ we can obtain the Schrödinger-type equation as

$$\psi'' + V\psi = 0\,. \tag{2.20}$$

The potential V takes the form:

$$V = -\frac{1}{4} M^2 a^2 \gamma e^{-z}(e^z - a^2\kappa_+)(e^z - a^2\kappa_-)\,, \tag{2.21}$$

where

$$\kappa_\pm = -\frac{2}{M^2a^4\gamma}\left\{1 \pm \sqrt{1 + \frac{M^4a^4\gamma}{4}\left(1 - \frac{a^4q}{R^4}\gamma\right)}\right\}. \tag{2.22}$$

There is also a turning point at $z = \ln(a^2\kappa_-)$. If the parameter γ satisfies the condition:

$$0 < \gamma < \frac{R^4}{2a^4q}\left\{\sqrt{1 + \frac{16q}{M^4R^4}} - 1\right\}, \tag{2.23}$$

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both of the parameters $\kappa_\pm$ become real (and positive) numbers. The condition (2.23) shows that the boost parameter γ is restricted within a certain range when $\frac{R^4}{2a^4q}\{\sqrt{1+\frac{16q}{M^4R^4}}-1\}$ is smaller than 1. The WKB approximation for this potential gives

$$\left(n+\frac{1}{2}\right)\pi=\sqrt{\frac{1}{4}M^2a^4\gamma}\int_0^{\kappa_-}dy\sqrt{\frac{(y-\kappa_+)(\kappa_--y)}{y^3}}\,, \tag{2.24}$$

where n denotes the integer. Here we have rewritten the WKB integral (2.24) by using the variable $y=a^{-2}e^z$. By substituting the variable $y=\kappa_-t$ into eq. (2.24), we can rewrite the WKB integral as

$$\begin{aligned}\left(n+\frac{1}{2}\right)\pi&=\sqrt{\frac{1}{4}M^2a^4\gamma\kappa_+}\int_0^1 dt\; t^{-3/2}(1-t)^{1/2}\times\\&\times\left\{1-\frac{1}{2}\frac{\kappa_-}{\kappa_+}t-\sum_{m=2}^{\infty}\frac{(2m-3)!!}{2^n}\left(\frac{\kappa_-}{\kappa_+}t\right)^m\right\}.\end{aligned} \tag{2.25}$$

In deriving eq. (2.25), we have expanded the expression $\sqrt{1-\frac{\kappa_-}{\kappa_+}t}$ in the Taylor's series by taking account of the fact that $0<-\frac{\kappa_-}{\kappa_+}t<1$.

The glueball masses are obtained by solving the eq. (2.25) for M. Up to the leading order in the parameter $\frac{\kappa_-}{\kappa_+}$ the 0^{++} glueball masses are given by

$$M^{0^{++}}_{(L)}=\frac{1}{a}\sqrt[4]{\frac{(2n+1)^2(2n-1)(2n+3)}{\gamma\left(1-a^4\gamma\frac{q}{R^4}\right)}}\,. \tag{2.26}$$

When the boost parameter γ satisfied the condition

$$0<\gamma<\frac{R^4}{a^4q}\,, \tag{2.27}$$

the mass spectrum (2.26) takes real numbers for positive integer $n=1,2,3,\ldots$ The condition (2.27) is stronger than the condition (2.23). The boost parameter γ is restricted by the instanton density q.

We next evaluate the glueball masses up to the subleading order in the parameter $\frac{\kappa_-}{\kappa_+}$. The mass spectrum is given by

$$M^{0^{++}}_{(L+SL)}=\frac{1}{a}\sqrt[4]{\frac{2f_+(n)}{9\gamma\left(1-a^4\gamma\frac{q}{R^4}\right)}}, \tag{2.28}$$

with the condition (2.27) and

$$\widetilde{M}^{0^{++}}_{(L+SL)}=\frac{1}{a}\sqrt[4]{\frac{2f_-(n)}{9\gamma\left(1-a^4\gamma\frac{q}{R^4}\right)}}, \tag{2.29}$$

with the condition $\frac{R^4}{a^4q}<\gamma$. Here $f_\pm(n)$ is the same function as (2.16). When we take the limit $q\to 0$, then the glueball masses (2.29) take the complex values and the states for 0^{++} glueball with the masses (2.29) becomes unstable. By virtue of the instanton effects,

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state	M (up to subleading)	$\widetilde{M}$ (up to subleading)
0^{++}	1 (input)	1 (input)
0^{++*}	1.89	0.75
0^{++**}	2.73	0.63
0^{++***}	3.56	0.56

Table 3: Masses of the 0^{++} glueball in NCYM in a constant self dual background.

the states for 0^{++} glueball with the masses (2.29) becomes stable. Although the instanton effects changes the 0^{++} glueball masses, they do not affect the ratio of the 0^{++} glueball masses. The ratios of the 0^{++} glueball masses $M^{0^{++}}_{(L+SL)}$ and $\widetilde{M}^{0^{++}}_{(L+SL)}$ in NCYM theory in a constant self-dual background are listed in table 3. The ratios of the glueball masses $\widetilde{M}^{0^{++}}_{(L+SL)}$ is different from that of the glueball masses $M^{0^{++}}_{(L+SL)}$.

3. The 1^{--} glueball masses in noncommutative gauge theory

We next evaluate the 1^{--} glueball masses in NCSYM theory in four dimensions using the dual supergravity description. The 1^{-+} and 1^{--} glueball masses can be derived from the 2-point function of the dimension 6 two-form operators $\mathcal{O}_6 = d^{abc} F_{\mu\rho}{}^a F^{\rho\sigma b} F_{\sigma\nu}{}^c$, where d^{abc} is the symmetric structure constant. The two-form operator $\mathcal{O}_6$ couples to the real part of a complex-valued antisymmetric tensor field $A_{\mu\nu}$ field which consists of the NS-NS and R-R two-forms fields. The operator contains 1^{-+} and 1^{--} components, which correspond to the fields A_{0i} and A_{ij}, where $i, j = 1, 2, 3$ correspond to the three coordinates of $\mathbb{R}^3$. When we evaluate the 1^{-+} and 1^{--} glueball masses, we have to solve the classical equation of motion of the massless antisymmetric tensor field $A_{\mu\nu}$ in the supergravity background [23].

Consider the wave equation for the complex-valued antisymmetric tensor field $A_{\mu\nu}$:

$$\partial_\mu \left\{ \sqrt{g} \partial_{[\,\kappa} A_{\rho\sigma\,]} g^{\mu\kappa} g^{\rho\nu} g^{\sigma\lambda} \right\} = 0 \,, \tag{3.1}$$

where the square brackets [ ] denotes antisymmetrization with the indices. Assuming the simplest ansatz we take only one component of the fluctuation. Under this assumption, the 1^{-+} or 1^{--} glueball mass spectrum depends on the components of the fluctuation, since the metric (2.1) is anisotropic in $\mathbb{R}^3$ due to the B-field background. First of all, we assume the only one component of the fluctuation A_{13} to be different from zero. The component A_{13} corresponds to the 1^{--} glueball.

We assume that the antisymmetric tensor field A_{13} is of the form $A_{13} = \psi(u) e^{ik\cdot x}$. Using the metric (2.1) one obtain the differential equation for A_{13}:

$$\partial_u \left[\, u\, \partial_u \psi(u) \,\right] + \left\{ \frac{M^2}{1-\beta^2} \frac{1}{u^3} \left(1 - \beta^2 (1 + a^4 u^4)\right) \right\} \psi(u) = 0 \,, \tag{3.2}$$

In deriving the wave equation (3.2), we have chosen a particular momentum $k^\mu = (\frac{M}{\sqrt{1-\beta^2}}, 0, \frac{\beta M}{\sqrt{1-\beta^2}}, 0)$. Changing the variables to $z = 2\ln(au)$, we have the Schrödinger type equation as

$$\psi'' + V_{13}^{1^{--}} \psi = 0 \,. \tag{3.3}$$

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Here $V_{13}^{1^{--}}$ denotes the potential

$$V_{13}^{1^{--}} = -\frac{1}{4} M^2 a^2 \gamma\, e^{-z} (e^z - a^2 \eta_+)(e^z - a^2 \eta_-)\,, \tag{3.4}$$

where

$$\eta_\pm \equiv \pm \frac{1}{a^2 \sqrt{\gamma}}\,, \tag{3.5}$$

with $\gamma \equiv \frac{\beta^2}{1-\beta^2}$. The WKB approximation for this potential can be rewritten by using the variable $y = a^{-2} e^z$:

$$\left(n + \frac{1}{2}\right) \pi = \frac{1}{2} M a^2 \sqrt{\gamma} \int_0^{\eta_+} dy \sqrt{\frac{(\eta_+ - y)(y - \eta_-)}{y}}\,, \tag{3.6}$$

where n denotes the integer. Carrying out of this integration, we obtain the mass spectrum for 1^{--} glueball:

$$M_{13}^{1^{--}} = \frac{1}{16a} \Gamma(1/4)^2 \sqrt[4]{\frac{(2n+1)^4}{\pi^2 \gamma}}\,. \tag{3.7}$$

As expected, the glueball masses are proportional to the inverse of the noncommutativity parameter.

In the next place, we assume the only one component of the fluctuation A_{03} to be different from zero. The component A_{03} corresponds to the 1^{-+} glueball.

The wave equation for A_{03} is given by

$$\partial_u \left[\, u\, \partial_u \psi(u)\, \right] - \left\{ \frac{M^2 \beta^2}{1-\beta^2} \frac{1 + a^4 u^4}{u^3} \right\} \psi(u) = 0\,. \tag{3.8}$$

In deriving the wave equation (3.8), we have set the dependencies $A_{03} = \psi(u) e^{ik\cdot x}$ and chosen a particular momentum $k^\mu = (\frac{M}{\sqrt{1-\beta^2}}, 0, \frac{\beta M}{\sqrt{1-\beta^2}}, 0)$. Under the change of variables to $z = 2\ln(au)$, one obtains the Schrödinger form of the equation:

$$\psi'' + V_{03}^{1^{-+}} \psi = 0\,. \tag{3.9}$$

Here $V_{03}^{1^{-+}}$ denotes the potential

$$V_{03}^{1^{-+}} = -\frac{1}{4} M^2 a^2 \gamma e^{-z} (1 + e^{2z})\,, \tag{3.10}$$

where $\gamma \equiv \frac{\beta^2}{1-\beta^2}$. Since the potential $V_{03}^{1^{-+}}$ takes negative value for all region of z, there is no turning point. Hence the WKB approximation for this potential $V_{03}^{1^{-+}}$ cannot lead the discrete mass spectrum for 1^{-+} glueball.

The remaining components which we should investigate are A_{01} and A_{23}. The components A_{01} and A_{23} correspond to the 1^{-+} and 1^{--} glueball, respectively. We assume that the antisymmetric tensor field $A_{\mu\nu}$ are of the form $A_{\mu\nu} = f(u) e^{ik\cdot x}$ and choose a particular momentum $k^\mu = (\frac{M}{\sqrt{1-\beta^2}}, 0, \frac{\beta M}{\sqrt{1-\beta^2}}, 0)$. For a suitable redefinition of the function $f(u)$ and the change of variable, we obtain the Schrödinger type equations for A_{01} and A_{23}.

state	$NCYM_4$ (WKB)	QCD_4 (WKB) [30]
1^{--}	1 (input)	1 (input)
1^{--*}	1.67	1.75
1^{--**}	2.33	-
1^{--***}	3.00	-

Table 4: Masses of the 1^{--} glueball in NCYM_4.

The corresponding potentials with the Schrödinger equations for A_{01} and A_{23} are given as follows,

$$V_{01}^{1^{-+}} = -\frac{1}{4}M^2a^2\gamma\, e^{-z}(1+e^{2z}) - \frac{e^{2z}}{1+e^{2z}}\left(2 - \frac{3e^{2z}}{1+e^{2z}}\right), \tag{3.11}$$

$$V_{23}^{1^{--}} = \frac{1}{4}M^2a^2(1+\gamma)\, e^{-z} - \frac{2e^{2z}}{1+e^{2z}} + \frac{e^{4z}}{(1+e^{2z})^2}, \tag{3.12}$$

respectively. These potentials have two turning points for a certain region in γ. Hence the WKB approximation for the potentials $V_{01}^{1^{-+}}$ and $V_{23}^{1^{--}}$ implies the discrete mass spectrum for 1^{-+} and 1^{--} glueball. For evaluation of these mass spectra, numerical approach is more useful than the WKB approximation. More detailed analysis will be shown in [31].

The ratios of the 1^{--} glueball masses obtained by solving the wave equation for anti-symmetric tensor field in the WKB approximation are listed in table 4. The supergravity results in finite temperature Yang-Mills theory in four dimensions are also listed in the same table.

From table 4, we find that the difference between the supergravity results in the QCD and the supergravity ones in the NCYM theory is not so large.

4. Conclusions and discussions

In this paper, we have evaluated the ratios of the glueball masses in large N NCSYM theories via the dual supergravity description. The mass spectrum of the scalar glueball 0^{++} and vector glueballs 1^{--} in noncommutative gauge theories are evaluated by solving the wave equations in the dual supergravity background.

In evaluating the mass eigenvalues, we have applied the WKB analysis to the supergravity wave equations. The WKB analysis exhibits that the mass spectrum for the glueball is discrete with a finite gap due to the space-space noncommutativity. The glueball masses in noncommutative gauge theories depend on the noncommutativity parameter a with dimension of length. The ratio of the glueball masses, however, does not depend on the noncommutativity parameter. These ratios are not so different from the non-supersymmetric model of QCD data. The supergravity dual of the noncommutative super Yang-Mills theory in a constant self-dual gauge field background is constructed by a certain limit of superposition of D3-brane and D(-1)-brane backgrounds. The effects of the constant self-dual gauge field background in noncommutative gauge theory can be estimated using the dual supergravity description. The constant self-dual gauge field background makes unstable glueball in noncommutative gauge theory stable with large D-instanton density q.

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The noncommutative gauge theories is not conformal due to the noncommutativity of space [17] and the space-space noncommutativity is reflected in some physical quantities in the noncommutative gauge theories. For instance, the Wilson loops in noncommutative gauge theory exhibit area law behavior for the noncommutativity effects are large. The string tension, which can be read off from the area law, is controlled by the noncommutativity parameter. Similarly, the discrete mass spectra of the glueball in noncommutative gauge theory are caused by the space-space noncommutativity. The glueball masses are also controlled by the noncommutativity parameter.

The glueball mass spectrum for ordinary $\mathcal{N} = 1$ super Yang-Mills theory within the Maldacena–Nùñez solution has been investigated and it has been shown that a discrete spectrum and a mass gap for glueball can be produced without any sort of cut-off [33]. It would be interesting subject to investigate the glueball masses in the noncommutative gauge theories with less than maximal supersymmetry from the noncommutative deformation of the Maldacena–Nùñez solution. We hope to discuss this subject in the future.

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